Full name(s): _____ .

Questions

1. Compute the tangent line to the curve $x^2 + y^3 + 2xy = 0$ at the point $(-1, 1)$ (first verify that the point is on the curve).

2. Compute the inverse of $f(x) = \frac{x+3}{x-1}$, and find $\frac{d}{dx} f^{-1}$ in two different ways, once using the quotient rule and once using the inverse differentiation rule. Verify that the results agree.

3. What is the maximum and minimum value of the following functions (state if one or both does not exist):

   - $I = [0, 1]$, $f(x) = x^2 - x^4$
   - $I = [0, 2]$, $f(x) = \frac{x}{x^2-2x}$
   - $I = [1/2, 2]$, $f(x) = x - 2\ln(x)$
   - $I = (1/3, 1]$, $f(x) = \cos(\pi x)$

4. A cylinder is inscribed in a cone of height H and base radius R. What is the maximum volume of the cylinder?

5. Use L'Hopital to compute the following limits:

   - $\lim_{x\to\infty} \frac{\ln(x)}{\ln(\ln(x))}$
   - $\lim_{x\to 0} \frac{1-\cos(2x)}{1-\cos(3x)}$
   - $\lim_{x\to 0^+} \frac{\sin(x)}{e^{\sqrt{x}}-1}$
   - $\lim_{x\to 0^+} \sin(x)\ln(\sin(x))$
   - $\lim_{x\to 0^+} x^{\sin(x)}$
   - $\lim_{x\to \frac{\pi}{2}} \frac{\ln(\cos(x))}{\tan(x)}$

6. **Challenge**. Show that $\lim_{x\to 0^+} x^{(x^x)} = 0$