Full name(s): ______________________________ .

Questions

1. Determine the radius of convergence of each of the following power series, as well as the behavior at the endpoints the radius of convergence is finite:

   (a) $\sum_{n=0}^{\infty} \frac{n^2}{n!} x^n$

   (b) $\sum_{n=0}^{\infty} \frac{1}{n} x^n$

   (c) $\sum_{n=1}^{\infty} \frac{1}{n^2} x^n$

   (d) $\sum_{n=2}^{\infty} \frac{x^n}{n \ln(n)}$

   (e) $\sum_{n=0}^{\infty} \frac{(1+n)^{2n}}{(1+n+2n^2)^n} x^n$

   (f) $\sum_{n=1}^{\infty} \frac{n^{3n}}{(3n)!} x^n$

   Determine the first three terms of the Taylor expansion for each of the following functions:

   1. $f(x) = \tan(x)$
   2. $f(x) = \ln(1-x)$
   3. $f(x) = \frac{x+1}{x-1}$ (hint: it may be helpful to write f using a partial fraction expansion first)
   4. $f(x) = e^{\sin(x)}$

2. Show via term by term integration of the series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ that in some interval around zero $\ln(1-x)$ has Taylor series:

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

3. Find the radius of convergence of the Taylor series for $\ln(1+x)$. What is the convergence behavior at the endpoints?