Full name(s): ________________________ .

Questions

1. Evaluate the following improper integrals:
   (a) $\int_0^\infty \frac{1}{1+2t^2} dt$
   (b) $\int_1^\infty \frac{\ln(x)}{x^2} dx$
   (c) $\int_{-\infty}^0 \frac{e^x}{1+e^x} dx$
   (d) $\int_0^\infty \frac{x}{e^x} dx$
   (e) $\int_0^\infty e^{-\sqrt{s}} ds$
   (f) $\int_{-\infty}^\infty \frac{1}{25+z^2} dz$
   (g) $\int_3^\infty \frac{1}{x \ln^2(x)} dx$
2. Given that $\int_{-\infty}^\infty e^{-x^2} dx = \sqrt{\pi}$, evaluate:

$$\int_{-\infty}^{\infty} e^{-(x-\mu)^2/\sigma^2} dx$$

3. The Γ function is defined for all $s \in (0, \infty)$ by

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$$

   (a) Find $\Gamma(1)$ and $\Gamma(2)$.
   (b) Use integration by parts to show that $\Gamma(n+1) = n\Gamma(n)$.
   (c) If n is a positive integer, what is $\Gamma(n)$?